

Aufgabe 1 (Sipser, exercise 2.1)

Es sei $G = (V, \Sigma, R, E)$ eine kontextfreie Grammatik mit $V = \{E, T, F\}$, $\Sigma = \{a, +, \times, (,)\}$ und den folgenden Regeln.

$$\begin{aligned} E &\rightarrow E+T \mid T \\ T &\rightarrow T\times F \mid F \\ F &\rightarrow (E) \mid a \end{aligned}$$

Geben Sie Parsebäume und Ableitungen für jeden der folgenden Strings an.

- (a) a
- (b) a+a
- (c) a+a+a
- (d) ((a))

Aufgabe 2 (Sipser, exercise 2.3, Teil (m) leicht modifiziert)

Answer each part for the following context-free grammar G :

$$\begin{aligned} R &\rightarrow XRX \mid S \\ S &\rightarrow aTb \mid bTa \\ T &\rightarrow XTX \mid X \mid \varepsilon \\ X &\rightarrow a \mid b \end{aligned}$$

- (a) What are the variables and terminals of G ? Which is the start variable?
- (b) Give three examples of strings in $L(G)$.
- (c) Give three examples of strings *not* in $L(G)$.
- (d) True or False: $T \Rightarrow aba$.
- (e) True or False: $T \xRightarrow{*} aba$.
- (f) True or False: $T \Rightarrow T$.
- (g) True or False: $T \xRightarrow{*} T$.

- (h) True or False: $XXX \xRightarrow{*} aba$.
- (i) True or False: $X \xRightarrow{*} aba$.
- (j) True or False: $T \xRightarrow{*} XX$.
- (k) True or False: $T \xRightarrow{*} XXX$.
- (l) True or False: $S \xRightarrow{*} \varepsilon$.
- (m) Give a description in German of $L(G)$.

Aufgabe 3 (Sipser, exercise 2.4, part a–c)

Give context-free grammars that generate the following languages. In all parts the alphabet Σ is $\{0, 1\}$.

- (a) $\{w \mid w \text{ contains at least three } 1\text{s}\}$
- (b) $\{w \mid w \text{ starts and ends with the same symbol}\}$
- (c) $\{w \mid \text{the length of } w \text{ is odd}\}$